

General Certificate of Education
June 2009
Advanced Level Examination



MATHEMATICS
Unit Further Pure 3

MFP3

Thursday 11 June 2009 9.00 am to 10.30 am

For this paper you must have:

- a 12-page answer book
 - the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP3.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The function $y(x)$ satisfies the differential equation

$$\frac{dy}{dx} = f(x, y)$$

where

$$f(x, y) = \sqrt{x^2 + y + 1}$$

and

$$y(3) = 2$$

(a) Use the Euler formula

$$y_{r+1} = y_r + h f(x_r, y_r)$$

with $h = 0.1$, to obtain an approximation to $y(3.1)$, giving your answer to four decimal places. *(3 marks)*

(b) Use the formula

$$y_{r+1} = y_{r-1} + 2h f(x_r, y_r)$$

with your answer to part (a), to obtain an approximation to $y(3.2)$, giving your answer to three decimal places. *(3 marks)*

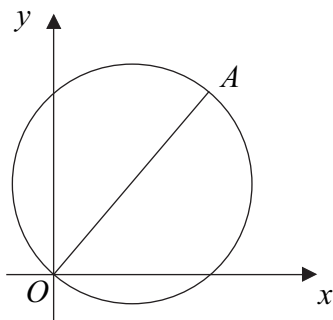
2 By using an integrating factor, find the solution of the differential equation

$$\frac{dy}{dx} - y \tan x = 2 \sin x$$

given that $y = 2$ when $x = 0$.

(9 marks)

- 3 The diagram shows a sketch of a circle which passes through the origin O .



The equation of the circle is $(x - 3)^2 + (y - 4)^2 = 25$ and OA is a diameter.

- (a) Find the cartesian coordinates of the point A . (2 marks)
- (b) Using O as the pole and the positive x -axis as the initial line, the polar coordinates of A are (k, α) .
- (i) Find the value of k and the value of $\tan \alpha$. (2 marks)
- (ii) Find the polar equation of the circle $(x - 3)^2 + (y - 4)^2 = 25$, giving your answer in the form $r = p \cos \theta + q \sin \theta$. (4 marks)

- 4 Evaluate the improper integral

$$\int_1^{\infty} \left(\frac{1}{x} - \frac{4}{4x+1} \right) dx$$

showing the limiting process used and giving your answer in the form $\ln k$, where k is a constant to be found. (5 marks)

- 5 It is given that y satisfies the differential equation

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = 8 \sin x + 4 \cos x$$

- (a) Find the value of the constant k for which $y = k \sin x$ is a particular integral of the given differential equation. (3 marks)
- (b) Solve the differential equation, expressing y in terms of x , given that $y = 1$ and $\frac{dy}{dx} = 4$ when $x = 0$. (8 marks)

Turn over ►

6 The function f is defined by

$$f(x) = (9 + \tan x)^{\frac{1}{2}}$$

(a) (i) Find $f''(x)$. (4 marks)

(ii) By using Maclaurin's theorem, show that, for small values of x ,

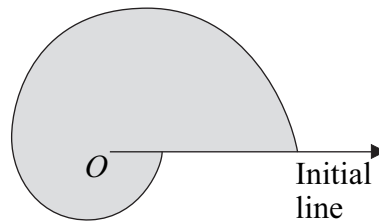
$$(9 + \tan x)^{\frac{1}{2}} \approx 3 + \frac{x}{6} - \frac{x^2}{216} \quad (3 \text{ marks})$$

(b) Find

$$\lim_{x \rightarrow 0} \left[\frac{f(x) - 3}{\sin 3x} \right] \quad (3 \text{ marks})$$

7 The diagram shows the curve C_1 with polar equation

$$r = 1 + 6e^{-\frac{\theta}{\pi}}, \quad 0 \leq \theta \leq 2\pi$$



(a) Find, in terms of π and e , the area of the shaded region bounded by C_1 and the initial line. (5 marks)

(b) The polar equation of a curve C_2 is

$$r = e^{\frac{\theta}{\pi}}, \quad 0 \leq \theta \leq 2\pi$$

Sketch the curve C_2 and state the polar coordinates of the end-points of this curve.

(4 marks)

(c) The curves C_1 and C_2 intersect at the point P . Find the polar coordinates of P .

(5 marks)

8 (a) Given that $x = t^2$, where $t \geq 0$, and that y is a function of x , show that:

(i) $2\sqrt{x} \frac{dy}{dx} = \frac{dy}{dt}$; (3 marks)

(ii) $4x \frac{d^2y}{dx^2} + 2 \frac{dy}{dx} = \frac{d^2y}{dt^2}$. (3 marks)

(b) Hence show that the substitution $x = t^2$, where $t \geq 0$, transforms the differential equation

$$4x \frac{d^2y}{dx^2} + 2(1 + 2\sqrt{x}) \frac{dy}{dx} - 3y = 0$$

into

$$\frac{d^2y}{dt^2} + 2 \frac{dy}{dt} - 3y = 0$$
 (2 marks)

(c) Hence find the general solution of the differential equation

$$4x \frac{d^2y}{dx^2} + 2(1 + 2\sqrt{x}) \frac{dy}{dx} - 3y = 0$$

giving your answer in the form $y = g(x)$. (4 marks)

END OF QUESTIONS

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